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Theory of Weak-Field Magnetoresistance in Graphene and Related Materials

The formula of weak-field magnetoconductivity $\Delta\sigma(B)$ proportional to B^2, with B the strength of a magnetic field, is derived based on the linear-response formula of the conductivity. In order to achieve expansion with respect to B, we first consider spatially varying magnetic field $B\cos(qx)$ with wavenumber q, then expand the Kubo formula of the conductivity with respect to B up to B^2, and finally expand the results with respect to q up to q^2 [1]. This is a straightforward extension of the scheme developed for the weak-field Hall conductivity [2-9] except that the procedure is much more complicated. Then, $\Delta\sigma(B)$ is represented by Feynman diagrams given by three hexagons in graphene systems, in which a matrix Hamiltonian contains terms linear in the wave vector.

Explicit calculations are performed within a self-consistent Born approximation for various kinds of scatterers in monolayer [10] and bilayer graphene [11]. The results show that the magnetoresistance $\Delta\rho(B) \propto B^2$ essentially vanishes away from the zero energy. This vanishing magnetoresistance is the result of the well-known cancellation with the counter term due to the Hall effect away from the zero energy. In the vicinity of zero energy, on the other hand, the magnetoresistance exhibits a sharp double-peak structure. This prominent feature arises due to the band crossing, i.e., an electron behaves partially as a negatively charged particle and also as a positively charged particle in the vicinity of zero energy. In monolayer graphene, the divergence of the classical cyclotron frequency $\omega_c \propto \varepsilon_F^{-1}$ also contributes to the enhancement of the double-peak structure.

This formula is extended to the case that the diagonal element of a matrix Hamiltonian contains a term proportional to $\boldsymbol{k}^2$, i.e., $\boldsymbol{k}^2/2m$ with mass m. Then, the result can be used in more general systems described by a $\boldsymbol{k}\cdot\boldsymbol{p}$ Hamiltonian based on the modified Bloch functions of Luttinger and Kohn [12]. The formula contains Feynman diagrams given by pentagons in addition to the hexagons in graphene systems [13]. It is used for calculation of singular magnetoresistance at the band crossing point of a two-dimensional system with a giant Rashba spin splitting.

References

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Figures

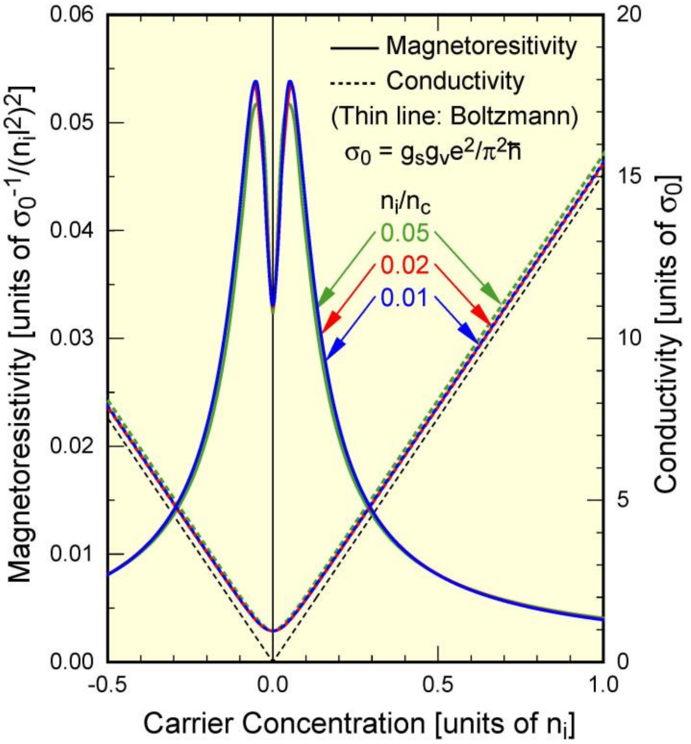

Figure 1: Calculated magnetoresistivity and zero-field conductivity in monolayer graphene in the case of dominant charged-impurity scattering. The carrier concentration is measured in units of impurity concentration *n*n_i. Corresponding Boltzmann results are denoted by thin lines [10].

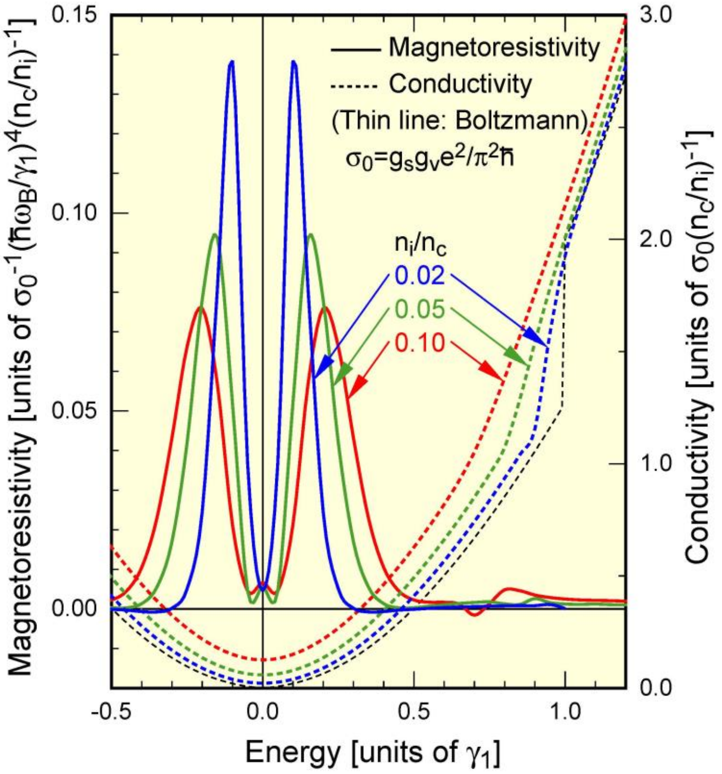

Figure 1: Calculated magnetoresistivity and zero-field conductivity in bilayer graphene in the case of dominant charged-impurity scattering. The energy is measured in units of the interlayer hopping integral γ_1. Corresponding Boltzmann results are denoted by thin lines [11].